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HEAT TRANSFER IN PACKED BEDS THROUGH WHICH WATER IS FLOWING

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Abstract—Heat-transfer measurements were carried out in packed beds through which water was flowing, for three sizes of glass spheres and one size of steel balls.

Experimental results for both the radially effective thermal conductivities and the overall heat-transfer coefficients were analysed by means of the theoretical equations, previously published by the authors, showing the close coincidence between the calculated and the observed data.

NOMENCLATURE

C_p, specific heat of fluid, kg-cal/kg degC;
D_p, diameter of solid, m;
D_t, inner diameter of container, m;
G, superficial mass velocity of fluid based on sectional area of bed, kg/m² h;
h_0, overall heat-transfer coefficient across the wall surface of packed bed, kg-cal/m² h degC;
h_w^0, apparent wall film coefficient with motionless fluid, kg-cal/m² h degC;
k_e, effective thermal conductivity in packed bed, in perpendicular direction to the overall fluid flow, kg-cal/m h degC;
k_e^0, effective thermal conductivity in packed bed with motionless fluid, kg-cal/m h degC;
k_f, thermal conductivity of fluid, kg-cal/m h degC;
k_s, thermal conductivity of solid, kg-cal/m h degC;
L, length of packed bed, m;
N_{Pe_m} $= N_{Pr} \cdot N_{Re_m}$;
N_{Pr}, $= C_p \mu / k_f$;
N_{Re_m} $= D_p G/\mu$;
R $= D_t/2$, radius of packed bed, m;
r, distance from the centre of cylindrical packed bed, m;
t, temperature, °C;
t_c, temperature on centre line of packed bed, °C;
t_0, temperature of inlet fluid, °C,
t_m, mean temperature across a sectional area, °C;
t_w, wall temperature, °C;
z, distance from entrance of packed bed, m;

Greek symbols
α, (mass velocity flowing in the direction of heat transfer)/(superficial mass velocity of fluid based on sectional area of empty container in the direction of fluid flowing);
α_w, (mass velocity of fluid flowing near wall surface in packed bed in the direction of heat transfer)/(superficial mass velocity of fluid based on sectional area of empty container in the direction of fluid flowing).
μ, viscosity of fluid, kg/m h;
ϵ, void fraction in the core portion of packed bed.

INTRODUCTION

In connexion with the design calculations of catalytic reactors, effective thermal conductivities in packed beds with flowing gases have been studied by a number of investigators [1–9], a summary of which can be found elsewhere [10, 11].

Applying a model of lateral mixing mechanism proposed by Ranz [12], Yagi and Kunii derived a theoretical equation of effective thermal conductivities in perpendicular direction to the flowing fluid in packed beds as follows [10, 11]:

$$\frac{k_e}{k_f} = \frac{k_e^0}{k_f} + (\alpha\beta)\, N_{Pr} \cdot N_{Re_m}. \qquad (1)$$

For stagnant thermal conductivity k_e^0, there have been also many studies in beds where the space surrounding the particle is filled with stationary fluid. Kunii and Smith [13], Willhite, Kunii and Smith [14] summarized the previous works and proposed models for heat transfer, obtaining theoretical equations of stagnant conductivities, which take into account the effects of thermal properties of solid and fluid phases. In equation (1), α means the ratio of the mass velocity of lateral mixing referring to the mass velocity based on the sectional area of the bed parallel to the overall direction of flowing fluid, β being the ratio of the average distance between centres of two particles touching each other. Equation (1) can explain satisfactorily almost all experimental data of radially effective thermal conductivities k_e analysed in such a way to avoid the effect of thermal resistance on the wall surface of the container. Examples of the experimental values of $\alpha\beta$ are shown in Fig. 1 as the function of the ratio of particle diameter D_p to bed diameter D_t, and the other data for different shapes of packings can be found elsewhere [11].

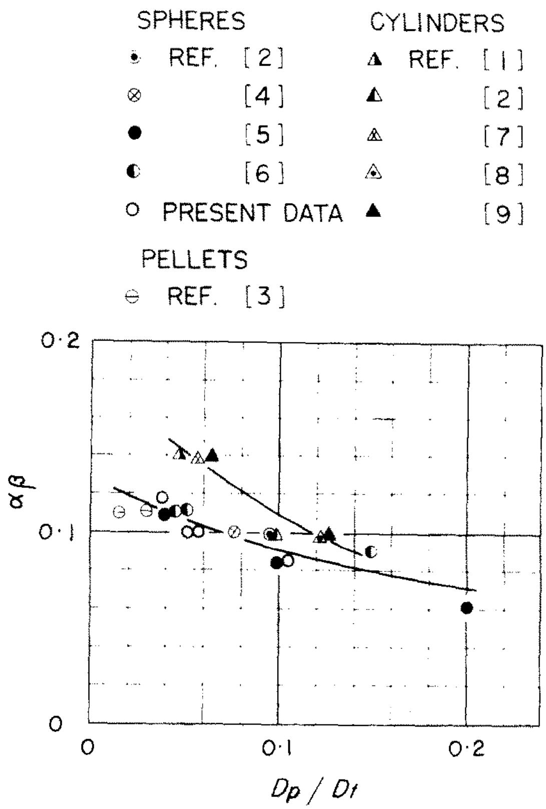

FIG. 1. Relation between $\alpha\beta$ and D_p/D_t, for ordinary cylindrical packed beds.

So far, the majority of experimental works have been done with the use of air, except two papers with natural gas [2] and with hydrogen, methane and propane [3], and no investigator has employed the liquid system for direct measurement of temperature distributions in packed beds with flowing liquid. Gopalarathnam, Hoelscher and Laddha [15] compared the above equation (1) with their analysed values of effective thermal conductivities in various liquid–solid systems, whereas their data being calculated from the experimental data of so-called overall heat-transfer coefficient by means of a procedure suggested by McAdams [16].

On the other hand, a number of investigations have been made to ascertain the relation between overall heat-transfer coefficients for packed beds and physical properties of the fluid stream as well as the physical dimensions of the apparatus, few correlations having been derived to cover almost all experimental data previously reported. Yagi and Kunii surveyed briefly on this subject in their original paper [17], and utilized the theoretical equations given by Hatta and Maeda [8] to compare their calculated values of overall heat-transfer coefficient h_0 with the experimental results previously published, including two prominent works for liquid–solid systems, namely by Kuzuoka with the water [18] and by Chennakesavan with toluene, nitrobenzene and aqueous glycerine [19].

In this paper, experimental results of both the effective thermal conductivities and the overall heat-transfer coefficients are presented for the packed beds through which water was flowing, and they are analysed numerically by means of theoretical equations previously presented by Yagi and Kunii.

EXPERIMENTAL APPARATUS AND PROCEDURE

Measurements were carried out for three sizes of glass spheres and one size of steel balls with the liquid water. The description of the solid particles and the operating conditions are summarized in Table 1. Figure 2 shows the details

Table 1. *Scope of experimental conditions and results*

		mean diameter D_p [mm]	void fraction ϵ
Solid	glass spheres	2.25	0·402
		3·02	0·416
		6·38	0·416
	steel balls	3·10	0·425
Water	inlet temperature	7·1 ~ 19·9°C	
	mass velocity	3·99 ~ 109 kg/m² h	
	N_{Re_m}	7·45 ~ 125	
	N_{Pr}	2·76 ~ 4·72	
	$N_{P\acute{e}_m}$	21·2 ~ 571	
	$(k_e/k_f)_{obs}$.	2·29 ~ 97·7	
	$(h_0 D_p/k_f)_{obs}$.	1·07 ~ 27·4	

of the 60 mm i.d. cylinder with steam jacket used as the packed bed. The water entered the top at slightly below room temperature and passed downward through the calming section composed of 4 mm glass beds for obtaining of the uniform velocity. In order to prevent the heating of water before entering the test section, the container of the calming section had the water cooling jacket. The packed bed was supported by the metal gauze which openings were almost 0·5 mm, and the heat loss from the exit part of the apparatus was minimized by means of sufficient insulation as shown in Fig. 2. Temperatures along the centre line of the bed were determined with six chromel–alumel thermocouples of approximately 0·3 mm o.d. Radial distribution of temperature was measured by the thermocouples extended 18 mm vertically upward from the outlet of the test section in order to prevent the error by thermal conduction through the lead wires. The longitudinal and radial locations of thermocouple junctions were checked very carefully before and after each run for one kind of packing. The water, overflowed from a head tank about 4 m high from the floor level, was introduced to the test section and measured after flowing out of it, by direct observation with measuring glass cylinders.

RESULTS

Numerical values of effective thermal conductivities k_e in perpendicular direction to the overall fluid flow could be calculated from the experimental data of temperature distributions in packed bed, by means of equations (2), (3) and (4).

For radial temperature distributions:

$$\frac{t_w - t}{t_w - t_c} \cong J_0 (a_1 \cdot r/R) \tag{2}$$

for longitudinal distributions along the centre line:

$$\frac{t_w - t_c}{t_w - t_0} \cong \text{const.} \cdot \exp(- a_1^2 \, y \, \eta) \tag{3}$$

for dimensionless groups:

$$y = \frac{k_e L}{G C_p R^2} \quad \eta = \frac{z}{L}. \tag{4}$$

Above equations are the solutions of partial differential equation for temperature at any radial and longitudinal position in packed beds through which fluid is flowing, and both the employed assumptions and the procedure of derivation can be found elsewhere [5]. From equation (3) one can get:

$$\ln \frac{t_w - t_c}{t_w - t_0} = \text{const.} - a_1^2 \, y \, \eta. \tag{5}$$

Consequently, numerical value of $a_1^2 y$ can be decided from a straight line in a plot of the logarithm of the temperature ratio against the depth ratio z/L, an example of which is shown in Fig. 3. From the experimental temperature distribution on the section 18 mm upward from the end of the packed bed, numerical value of $a_1 \cdot r/R$ was obtained with the aid of equation (2) for each radial position r, and then the value of a_1 was decided as the slope of straight line for $a_1 \cdot r/R$ vs. r/R plot. Figure 4 shows comparatively good linearity of the data correlated as above mentioned, proving the adequacy of the present

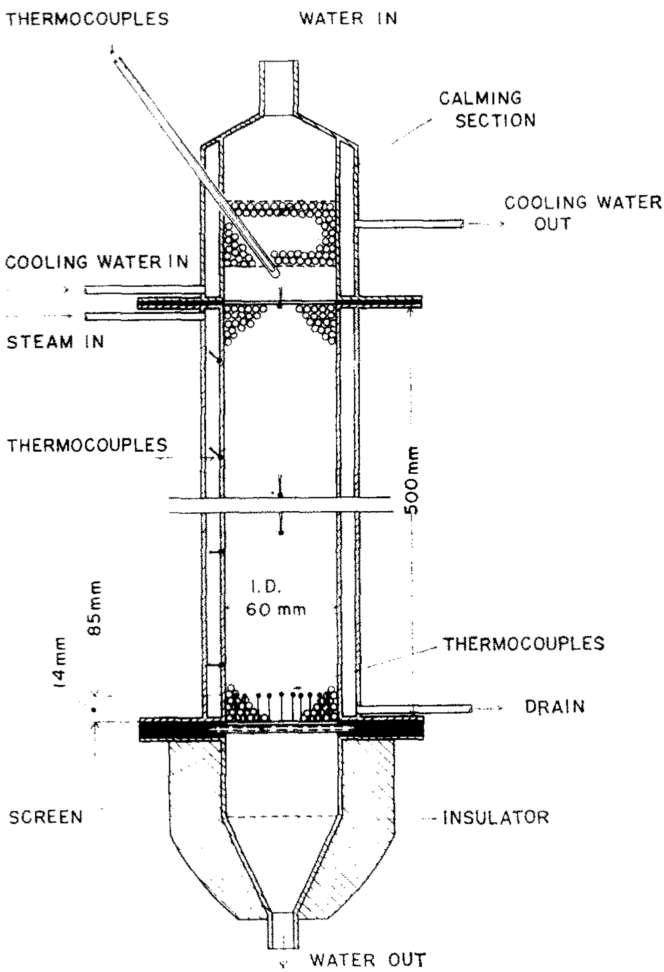

FIG. 2. Details of test section.

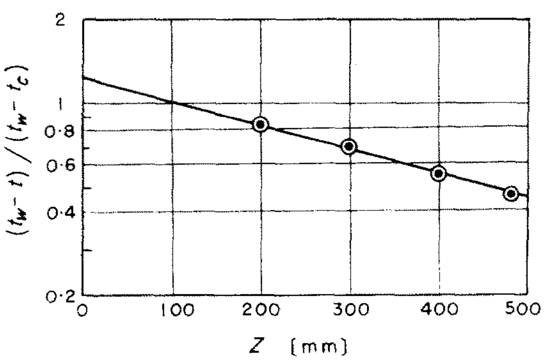

FIG. 3. Plot of logarithm of temperature ratio against depth z.

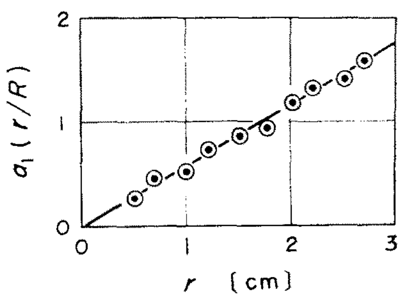

FIG. 4. $a_1(r/R)$ vs. r plot.

procedure. Knowing the values of both $a_1^2 y$ and a_1, effective thermal conductivity k_e can be decided easily as follows:

$$\frac{k_e}{k_f} = \frac{G\,C_p\,R^2 y}{L\,k_f} = \frac{1}{4}\left(\frac{D_t}{D_p}\right)\left(\frac{D_t}{L}\right)(N_{Pr}\cdot N_{Re_m})\frac{a_1^2 y}{(a_1)^2}. \tag{6}$$

Experimental data thus obtained are shown in Fig. 5, indicating the theoretical equation (1) is also adequate in liquid–solid system, where two values of both k_e^0/k_f and $\alpha\beta$ should be taken as shown in Table 2.

Table 2. Results obtained from experimental data

Solids	Glass spheres			Steel balls
D_p [mm]	2·25	3·04	6·38	3·10
D_p/D_t	0·0375	0·0504	0·106	0·0517
k_e^0/k_f*	1·47	1·47	1·47	7·87
$\alpha\beta$†	0·115	0·11	0·09	0·11

* Calculated from theoretical equations by Kunii and Smith [13], taking $k_s = 0{\cdot}8$, $k_f = 0{\cdot}56$ kcal/m h degC and $\epsilon = 0{\cdot}40$.

† Determined by the slope of straight line.

The numerical values of $\alpha\beta$ above obtained were plotted in Fig. 1 against the ratio D_p/D_t as the abscissa in order to compare with the values for gas–solid systems. It should be noted that there would be no significant difference between $\alpha\beta$ values for both gas–solid and liquid–solid systems respectively, indicating the adequacy of the proposed mechanism of lateral mixing in packed beds. Conclusion by Gopalarathnam, Hoelscher and Laddha [15] suggested the similar consistency to the data above mentioned. Considering above results, it seems quite reasonable to employ the averaged lines in Fig. 1 for interpolation of $\alpha\beta$ values in cases of any different fluid, as long as the Prandtl number of the fluid being between those of the air and the water. Approximately, it might be possible to apply the values of $\alpha\beta$ determined from the experimental data in packed beds of different shape of packings through which the air was flowing, in order to estimate these values for the different kind of fluid. Three convenient charts for $\alpha\beta$ values of Raschig rings, Berl saddles and broken solids were presented elsewhere [5, 11].

On the other hand, the numerical values of overall heat-transfer coefficient h_0 can be calculated from equation (7), employing the weighted mean temperature t_m along the radius with the assumption of rod-like flow pattern.

$$(\pi D_t\,L)\,h_0\,\frac{(t_w - t_0) - (t_w - t_m)}{\ln[(t_w - t_0)/(t_w - t_m)]} = \left(\frac{\pi}{4}D_t^2\right) G\,C_p\,(t_m - t_0). \tag{7}$$

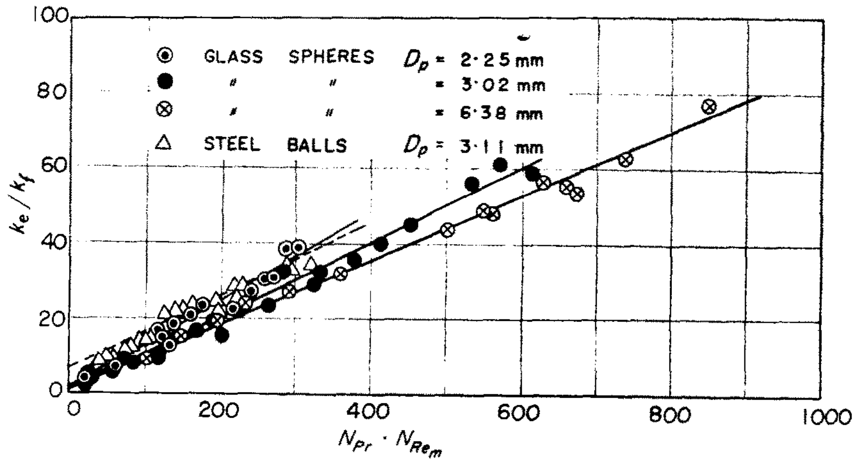

FIG. 5. Experimental data of k_e/k_f plotted against modified Pèclèt number $N_{Pe_m} = N_{Pr}\cdot N_{Re_m}$.

Hence,

$$N_{Nu} = \frac{h_0 D_p}{k_f} = \frac{1}{4} N_{Pr} \cdot N_{Re_m} \left(\frac{D_t}{L}\right) \ln \frac{t_w - t_0}{t_w - t_m}. \quad (8)$$

Experimental data thus obtained are plotted in Fig. 6 against the modified Pèclèt number $N_{Pè_m} = N_{Pr} \cdot N_{Re_m}$.

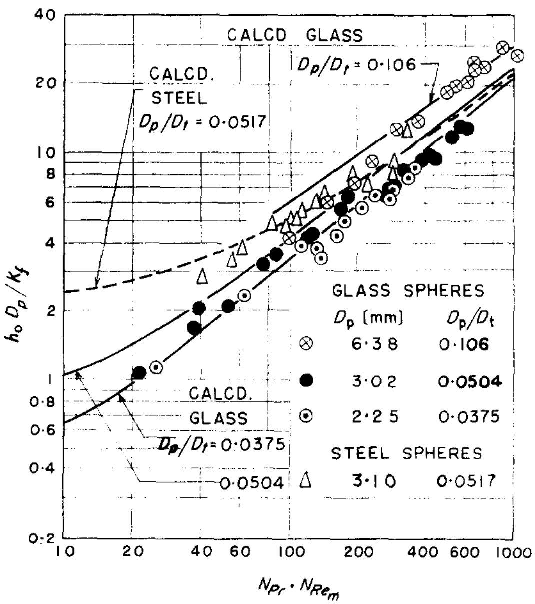

FIG. 6. Comparison of calculated values of $h_0 D_p/k_f$ with experimental data.

COMPARISON OF CALCULATED AND OBSERVED OVERALL HEAT-TRANSFER COEFFICIENTS

Utilizing the theoretical solution by Hatta and Maeda [8], Yagi and Kunii [17] presented a reasonable procedure for the estimation of overall heat-transfer coefficients h_0 in packed beds.

Applying the above procedure to the present system with flowing water, theoretical values of overall heat-transfer coefficients were calculated and compared with the experimental data as shown in Fig. 6. During the calculation, the following two values were needed, namely the ratio of mass velocity of lateral mixing near wall surface referring to the overall mass velocity based on the sectional area a_w, and then the Nusselt number $h_w^0 D_p/k_f$ with respect to the apparent heat-transfer coefficient h_w^0 on wall surface without fluid flowing, which were taken as follows:

$$a_w = 0{\cdot}054 \text{ estimated, [17]}$$

$$\frac{h_w^0 D_p}{k_f} = 17{\cdot}9 \text{ calculated for glass spheres}$$

$$= 12{\cdot}4 \text{ calculated for steel balls.}$$

The agreement between the computed Nusselt numbers and observed ones, is good as shown in Fig. 6 in the all range of modified Pèclèt number. For the low values of modified Pèclèt number, the contribution of thermal conductivities of solid phase is apparent, whereas becoming negligible for high Pèclèt number. Identical conclusions have been obtained by Yagi and Kunii (17), by comparison of their evaluated values with the experimental data previously reported by several investigators not only for liquid–solid systems but also for gas–solid systems.

CONCLUSION

(1) The model of lateral mixing of fluid in packed beds seems adequate in liquid–solid systems, similarly to the gas–solid systems. The numerical data of $\alpha\beta$ are plotted as shown in Fig. 1, which could be utilized generally for estimation of the effective thermal conductivities in packed beds through which any kind of fluid is flowing.

(2) Overall heat-transfer coefficients are observed in the present system. Agreement between the theoretical calculation by means of Yagi and Kunii's procedure and the observed data indicated the adequacy of the heat-transfer mechanism in packed beds proposed by the above authors.

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Résumé—On a conduit des mesures de transfert de chaleur dans des lits fixes à travers lesquels s'écoule de l'eau, ceci pour trois dimensions de bille de verre et un dimension de bille d'acier.

On a analysé les résultats expérimentaux pour les conductivités thermiques effectives radiales et les coefficients globaux de transfert de chaleur au moyen des équations théoriques publiées précédemment par les auteurs, en montrant le bon accord entre les valeurs calculées et observées.

Zusammenfassung—In wasserdurchströmten Festbetten aus Glaskugeln dreier verschiedener Grössen und aus Stahlkugeln einer Grösse wurden Wärmeübergangsmessungen durchgeführt.

Versuchsergebnisse sowohl für die wirksame Wärmeleitfähigkeit quer zur Hauptströmungsrichtung, als auch für die mittleren Wärmeübergangskoeffizienten wurden mit Hilfe kürzlich von den Autoren veröffentlichter theoretischer Gleichungen analysiert. Dabei ergab sich gute Übereinstimmung zwischen berechneten und beobachteten Werten.

Аннотация—Производились измерения теплообмена плотных упаковок, через которые протекает вода, для стеклянных шариков трех размеров и стальных одного размера.

Экспериментальные данные как для радиально-эффективной теплопроводности, так и для общих коэффициентов теплопередачи анализировались с помощью теоретических уравнений, опубликованных авторами ранее, что показало близкое совпадение расчетных данных с экспериментальными.